Surreptitious structure of the Turkish financial crisis 2000-2001

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Abstract

In this paper we develop a comprehensive framework to identify and analyze the non-marketable public debt stock accumulated by the informal debt-creating transactions among public sector entities, an important share of debt stock of public sector in Turkey that is backed by special budgetary rules with significant time lag and is hidden from public surveillance. This framework consists of three parts: Descriptive, modelling and empirical. By determining the “accurate” budget process of public sector and its “invisible balance”, we find out the hidden non-marketable debt stock data from 1989Q1 to 2009Q4. Then, we develop a dynamical model to seek whether Treasury’s process of servicing the non-marketable public debt results with the contraction of liquidity in public sector, leading to the financial crisis 2000-2001. Finally we analyze the hidden debt stock data on phase-space and confirm the findings of our model, which shows that the debt dynamics may follow a non-linear path.

JEL classification: F53; H63; H83

Keywords: lack of fiscal transparency; informal debt-creating transactions; non-marketable public debt stock; public debt crisis dynamics; phase-space reconstruction; max Lyapunov exponent; Turkey; IMF

1. Introduction

The literature on non-marketable public debt stock mainly focuses on market value estimates for outstanding government debt including non-marketable debt items in order to obtain total marketable government debt, Seater (1981), Cox and Hirschorn (1983), Butkiewicz (1983) and Marcet and Scott (2009). These studies on US government debt stock enjoy the advantageous of having complete information on the amount of securities and the main complication only arises in finding a consistent series for the market value of debt. On the other hand, for the developing economies because of lack of transparency the analysis of government debt is generally ill-posed even for multi-lateral institutions. In this context, the international creditors can probably be misinformed. For IMF this is crucial since in 90’s several countries have suffered from the simultaneous onset of currency crises and banking crises leading to full-blown economic crises while their economies were under monitoring of IMF (Stiglitz et al.,1998). It is well known that debt crises can be caused by political, institutional and structural

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problems that may well have existed for an extended period or have been deteriorating over time but that have so far been unobserved or undetected by international investors (Dreher et al., 2006). Moreover, financial markets are always subject to self-fulfilling expectations; if they believe that things will go wrong, things go wrong (Wyplosz, 2007). Any sunspot event could trigger speculators to gather new relevant information and/or to reassess existing information, so that they refuse to rollover maturing debt, withdraw their capital, or at least demand very high interest rates (Calvo, 2009). By doing so, they impose high pressure on interest rates and aggravate (sovereign) borrowers’ financial problems at the same time (see e.g. Calvo, 1988; Alesina et al., 1990; Detragiache, 1996; Cole and Kehoe, 2000). From this perspective, an economic programme established by IMF which addresses the debtor country’s deteriorating fundamentals inevitably does not work if the released data for such fundamentals are misleading. In this sense Turkish experiment¹ with crisis episodes 1994 and 2001 is an appropriate example and may shed light on the mechanism of recent debt crises across Europe (see Tamgac, 2011). There are other indebted countries in Europe and the fear is that a default on Greece can be a trigger for a wave of speculative attacks on other potential defaulters, such as Portugal, Spain, Italy and England an action likely to be self-fulfilling and massively damaging to the stability and prestige of the currency².

The plan of the paper is as follows. Section 2 examines Turkish public post-liberalization debt stock data and documents a number of stylized facts on the structural properties of debt mechanism. Section 3 identifies the “invisible balance” of public sector in Turkey and introduces its budget identities. Section 4 begins our theoretical analysis of the public debt by examining Treasury’s net-borrowing in terms of borrowing instruments and of its usage. The quarterly period for analysis is from 1989Q1 to 2009Q1. It reveals a substantial amount of net-borrowing from market which persistently exceeds the borrowing requirement of public sector, what we call as “additional net-borrowing” or “extra net-borrowing” interchangeably. Section 5 develops an unobserved non-marketable debt accumulation model. The model distinguishes between two cases depending upon how the cost-of-extra net borrowing will be financed. Within finite periods before the payment process is accomplished, the model then seeks whether the amount of extra net-borrowing reaches such a value which results in high magnitude illiquidity coinciding with the divergent character of the attractor. In Section 5, by applying our model we turn to the debt crises in 1994 and 2001 to quantify the importance of these effects vis a vis the

² To prevent such a self-fulfilling speculative attack on the currency Eurozone has agreed on a rescue package of €110 billion over the next three years including assistance from the International Monetary Fund as well as bilateral loans from the euro area countries. For how self-fulfilling expectations play an important role in the spread of crises across countries, i.e. contagion, see Goldstein and Pauzner (2004), Guimaraes and Morris (2004) and Keister (2009).
stylized facts discussed in Sections 2 and 3. Section 5 finally predicts the magnitudes of illiquidity associated with Turkish debt crises in 1994 and 2001, respectively. Since Treasury’s additional net-borrowing process is an implication of a deterministic fiscal policy, Section 6 provides a broader characterization of debt dynamics, and examines the time evolution characteristics of additional net-borrowing data by implementing dynamical systems theory, particularly by reconstructing phase-space and computing maximal LE. The motivation behind the analysis of attractor(s) on phase-space is to locate an unpredictable outcome (shift in equilibria set) of such borrowing process whose initialization is rather deterministic.

In IMF Report No:02/55 ROSC (March 2002) for the scope of government finance statistics of Turkey, it is stated (p.16):  “Very detailed government finance statistics are published for the budget sector. In addition, less detailed statistics, incorporating data for the nonbudget sector, are published in a variety of documents relating to macroeconomic issues and to the budget planning process. However, there is no single place in which to find complete and comprehensive government finance data.”

2. Informal debt-creating (non-budgetary) transactions and non-marketable public debt stock

The public sector in Turkey comprises eight main public entities, each of which has its own budget according to budget appropriation laws (see IMF report no:00/14 2000). These are Central Government (CG), Local Authorities (LA), State Economic Entreprises (SEE), Social Security Institutions (SSI), Extra Budgetary Funds (EBF), Unemployment Insurance Fund (UF or UI), the SEEs under privatization, and the Central Bank (CBRT). The SEE comprises financial SEEs (state banks) and non-financial SEEs. The public debt stock in Turkey is composed of external and domestic debt stocks of entire public sector. Beginning from 1986, the external debt stock of each public sector entity is quarterly announced by Treasury. The external debt stock data are sufficiently detailed, transparent and consistent (Önel and Utkulu 2006). Even though the domestic debt stock data of CG are announced, those of OG are not announced. The domestic debt stock of public sector should be considered as the union of marketable domestic debt stock and “non-marketable domestic debt stock” which we refer simply non-marketable debt stock. By definition the marketable debt stock of public sector is securitized and it accumulates by the issuance of cash governmental bonds (G-

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3 Beginning from 2004, the net debt stock of public sector is published annually.
4 Marketable securities can only be issued by Treasury and on behalf of CG, but not of OG. Debt management commission, Dikec, SPO (2001); Treasury Operations Report 1998 and 1999, Turkish Court of Accounts.
bonds) and cash treasury bills (T-bills) to the financial markets. The data for the marketable debt stock can be found on periodical announcements of Treasury. On the other hand, the non-marketable debt stock consists of the liabilities of 8 public sector entities to each other, implying that any entity may become debtor or creditor or both within public sector’s fiscal operations network. The non-marketable debt is completely domestic and is only held by creditor public entities and is closed to public trade. The non-marketable debt stock accumulates from the informal debtor/creditor relationships among public sector entities. The informal debtor/creditor relationships are established by transactions which are not registered to transactors’ budgets. The non-budgetary transactions can occur in form of cash transactions and/or in form of transactions of goods and/or services both commercial and noncommercial. More precisely, non-budgetary transactions are debt-creating transactions among public entities, which are informal and unrecorded unless they are securitized. These type of transactions are bidirectional and take place on one hand, between CG and each OG on the other hand, among each OG. Those transactions will be included budget appropriation laws as long as they are securitized, but neither are registered in associated budget revenues/expenditures/transfers nor in budget financing items. In the literature there are related concepts such as extrabudgetary activities (EBA) (see Allen and Radev 2006; IMF Manual p.52) and quasi-fiscal activities (QFA) (see IMF Manual pg.16) overlapping the term “non-budgetary transactions”. The non-budgetary transactions differ from both EBAs and QFAs in various respects; first, non-budgetary transactions may be included in budget appropriation laws, second they do not include revenues and expenditures, third they are not limited by the transactions of general government or transactions of public corporations on the behalf of general government, and finally, their accumulation creates a debt stock, which is not marketable. The non-budgetary transactions are not observed on the related public entities’ budgets and thus are hidden transactions. The non-marketable debt stock may or may not be securitized, or a part of it may be securitized. This type of securitization (if it is done) takes place when Treasury issues “non-cash” governmental bonds (non-cash G-bonds) and “non-cash” treasury bills (non-cash T-bills) to the public entities which are determined to be creditor following the consolidation of non-budgetary transactions. This securitization leads to an increase in both non-cash G-bond and non-cash T-bill stocks. The stock of the non-cash securities is referred “securitized non-marketable public debt stock”. At the

\footnote{5 The terms “hidden transactions” and “non-budgetary transactions” can be used interchangeably throughout the paper.}
end of each fiscal year, the annual change in securitized share of non-marketable debt stock becomes publicly observable. Likewise, the unsecuritized share of non-marketable public debt stock is referred “unsecuritized non-marketable public debt stock”. Therefore the “accurate” debt stock of entire public sector is composed of marketable public debt stock, securitized non-marketable public debt stock and unsecuritized non-marketable public debt stock. Treasury performs non-cash borrowing by issuing special type G-bonds to creditor public sector entities. The non-cash borrowing is done according to consolidation law, CBRT law and budget laws constituted in the beginning of each fiscal year. By implementing those laws, mainly 6 types of special non-cash G-bonds and T-bills are issued for different usages (see Dikec 2001). First, Holding G-Bonds are issued for the debts of OG to CG. Second, Consolidation G-Bonds are issued for the debt of OG to OG. Third, the alteration of FX/TL value leads alteration in external debt stock and the Treasury finances the FX differences by issuing non-cash G-Bonds to CBRT in correspondence of cash receipt. Fourth, for budget financing Treasury is also able to borrow cash from CBRT and issues Short-term cash advances (STA) G-bonds to CBRT. Fifth, the OG run budget deficit because of Treasury’s lagging repayments. The unsecuritized debt of CG to OG is classified as Duty loses mechanism. Eichengreen (2001) point out that in 1999 duty loses reach %15 of GDP and the main part of these duty loses is unsecuritized. Finally, Interest payments can be converted to future principal payments. That is when the coupons of the above-listed 5 special type non-cash securities are matured, Treasury may not pay and instead may issue further special type of bonds converting the matured interest payments to future principals. Therefore we observe that the announced data depict only a share of the exact borrowing requirement of public sector (PSBR). In report 02/55 p.48 (2002) the IMF states “...Government finance statistics cover the whole of the general government sector, but nonbudgetary government finance statistics are only available annually, and with a significant time lag”. This remark of the IMF clearly shows that the IMF is informed of CG budget data, CG debt stock statistics and the PSBR data, however Turkish authorities do not present the data on the informal debt-creating transactions and the interest payments associated with them. Therefore, a significant part of net-increase in annual/quarterly public debt stock is hidden from the public surveillance.

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7 The interest rate for the securities issued for duty loses of State Banks is generally determined by consensus between Treasury and State Banks such that the budget(s) of State Bank(s) apparently run cash surplus, it in fact has cash deficit, see Turkish Court of Accounts report 1998.
3. Public sector budget structure and its “accurate” financing requirement

This section presents the hidden part of public sector’s budget structure which produces the non-marketable debt stock and introduces the budget identities characterizing the “accurate” budget structure of public sector in Turkey. Figure 1 depicts the “accurate” budget structure of public sector. To determine “accurate” financing requirement of public sector, the balance of public sector’s budget should be decomposed into its distinct items. These are balance for budgetary transactions and balance for non-budgetary transactions. The balance for budgetary transactions are shown by “Budgetary balance” and is composed of cash requirement of CG’s budget and that of OG’s aggregate budgets. On the other hand, the debtor/creditor relationships among public entities through non-budgetary transactions give rise to another type of balance: “Non-budgetary balance”. Since this balance is unregistered we can consider it “Invisible balance”. The non-budgetary balance is composed of two distinct items: i.) Balance of non-budgetary and Securitized transactions, ii.) Balance of non-budgetary and Unsecuritized transactions. The analytical expressions summarizing Figure 1 are proposed\(^8\) by Eqs. (A.1) to (A.12) in Appendix A.

Figure 1 “Accurate” budget structure of Turkish public sector and its financing

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\(^8\) In order to avoid notational complexity the budget identities are expressed in terms of “words”.

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Focusing on the balance of CG underestimates the extent of the deficit of the entire public sector, as was the case in some years, when the deficit of the rest of the components of the public sector was manifold that of
the CG. Despite this fact, the studies on Turkish public debt stock have focused only on the debt stock of CG (see Akcay et al., 2001; Keyder 2003; Jobert et al., 2007; Payne et al., 2008), leading to underestimations.

4. Treasury’s additional net-borrowing

According to the literature Eq.(1) should hold for public sector overall budget, where LHS of Eq.(1) denotes the difference between received and paid principals from market, $G$ denotes the non-interest spendings, $T$ denotes the revenues, $IP$ denotes interest payments and hence $G-T$ denotes the primary balance of total public sector.

$$\left[B_{t}^{(r)}-B_{t}^{(p)}\right]=\left(IP\right)_{t}+G_{t}^{p}-T_{t}^{p}$$

(1)

However, we consider that Eq.(1) has an implicit assumption. That is, the net-change in gross debt should be equal to the overall budget deficit. On the other hand we observe that this is impossible or hardly possible because of the transparency policy of Treasury and that the non-budgetary transactions. Figure 2 clarifies the issue and depicts the series obtained by

$$\frac{\left[B_{t}^{(r)}-B_{t}^{(p)}\right]-\left(IP\right)_{t}+G_{t}^{p}-T_{t}^{p}}{GDPA_{t}}$$

It is easy to see that there exists substantial difference between net-increase in announced nominal debt stock and borrowing requirement of total public sector per annual nominal GDP. The series signifying that difference has positive mean, implying persistent unobserved deficit of public sector and hence non-marketable debt stock. Therefore let us rewrite Eq.(1) as given in Eq.(2), where in nominal terms $e_{t}$ denotes the Treasury’s additional net-borrowing from market and measures a partition of non-marketable debt service which is paid to non-cash G-bond holders within a given period $T$. The net-increase in announced debt stock should be equal to the sum of budget financing of public sector (budgetary transactions) and payments for securitized part of non-marketable debt stock. Therefore, we have to revise the general budget identity given by Eq.(1).

$$\left[B_{t}^{(r)}-B_{t}^{(p)}\right]=\left(IP\right)_{t}+G_{t}^{p}-T_{t}^{p}+e_{t}$$

(2)

The term $e_{t}$ reveals the payment process of non-marketable debt stock. The $e_{t}$ data can directly be obtained from Eq.(2) and takes place as the second term in RHS of (Eq.6 appendix). Figure 2 shows that from 1989Q1 to 2009Q1 Treasury performs additional net-borrowing fluctuating around 6.5 % GDP which nearly

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9 The stock data can be found in Debt Statistics released by Treasury www.treasury.gov.tr and also Annual Debt Management Reports released by Treasury from 1999 to 2008. The data can be supplied upon to request.
disappears beginning from 2007Q1\(^{10}\). This amount is used to pay instalments of securitized non-marketable debt stock. The next section deals with the issue in details.

**Figure 2.** The difference between net-increase in nominal debt stock and borrowing requirement per nominal annual GDP.

![Figure 2](image_url)

We have to note that the two main spikes shown in Figure 2 with 25% and 30% magnitudes, signify liquidity crises occurred in 1994Q2 and 2001Q2, respectively. On the other hand, in 2001Q1 and 2001Q3, there exists further two spikes with magnitudes 14% and 14.5%, respectively. Figure 2 enables us to detect the hidden deficit which has accumulated within public sector fiscal operations network.

5. **A model for the non-budgetary transactions and possible debt crisis**

This sub-section proposes a modelling for both the accumulation of hidden transactions and the payment process of the securitized non-marketable debt stock in order to seek whether illiquidity in public sector originates from the dynamical behavior of the payment process. Following from the information stated in section 2, let us assume that at the end of period \(T-1\) Treasury decides to securitize some amount of non-marketable debt stock by negotiating the interest rate on non-cash G-bonds with OG and that schedules to pay the debt stock and associated interests within successive \(K\)-periods. Let us denote the securitized amount of non-marketable debt stock by \(S_T\) and its value in real terms by \(S_T'\), where \(S_T' = \frac{S_T}{P_T}\) and \(P_T\) is price level.

To pay each principal \(g_{T+k}\) in every period \(k, 1 \leq k \leq K\), Treasury borrows the nominal additional net-amount \(e_{T+k}\) from market with nominal interest rate, \(i_{T+k}\). Let \(E_{T+k}'\) and \(g_{T+k}'\) denote the values of \(e_{T+k}\)

\(^{10}\) For the detailed analysis the reader is referred to Debt Management Report 2008, Treasury, www.treasury.gov.tr
and \( g_{T+k} \) in real terms, respectively. In a given period \( k \), the payment to non-cash G-bond holders \( g_{T+k}^{r} \), may or may not be equal to the amount of additional net-borrowing \( E_{T+k}^{r} \) depending upon how the future interest payments associated with \( E_{T+k}^{r} \) will be financed. Our model captures all possibilities regarding the source for financing those interest payments. More specifically, what we do is to obtain first; the time evolution of \( S_{T}^{r} \) through \( k \)-periods. We assume that at period \( T \), Treasury initializes the payment process by paying the principal \( g_{T}^{r} \). In every subsequent period \( k \), in addition to principal payment \( g_{T+k}^{r} \), Treasury pays both the interest associated with \( S_{T}^{r} \) to OG and the interest for additional net-borrowing \( E_{T+k}^{r} \) to the market. Secondly, we seek that at the end of period \( T+k \), how much stock \( S_{T}^{r} \) rests to pay with associated interest payments and that to how extent this amount can affect the public budget balance: Is it lower enough to deal with or great enough to cause any liquidity crisis? That is for some \( k \), at period \( T+k \) if the government is forced endogeneously or exogenously to pay rest of all non-cash securities and associated interest, how much \( E_{T+k}^{r} \) Treasury should borrow from market? During the payment process one may assume that the non-budgetary transactions among public sector entities still continue. Let us denote this amount in real terms as \( \Delta_{T+k,T+k-1}^{r} \) within any period \( T+k \). The real interest rate on non-marketable debt stock is shown by \( i_{T}^{r*} \) and real interest rate for \( E_{T}^{r} \) is shown by \( i_{T}^{r} \). Consider the dynamical process \( \Omega \) shown below.

<table>
<thead>
<tr>
<th></th>
<th>Non-marketable Debt stock</th>
<th>Non-budetary transactions</th>
<th>Principal Payment of ( S_{T}^{r} )</th>
<th>Interest Payment for ( E_{T+k}^{r} )</th>
<th>Receipt from market</th>
</tr>
</thead>
<tbody>
<tr>
<td>At period ( T )</td>
<td>( S_{T}^{r} )</td>
<td>0</td>
<td>( g_{T}^{r} )</td>
<td>0</td>
<td>( E_{T}^{r} )</td>
</tr>
<tr>
<td>( T+1 )</td>
<td>( (1+i_{T+1}^{r*})S_{T+1}^{r} )</td>
<td>( \Delta_{T+1,T}^{r} )</td>
<td>( g_{T+1}^{r} )</td>
<td>( f_{T+1}^{r} )</td>
<td>( E_{T+1}^{r} )</td>
</tr>
<tr>
<td>( T+2 )</td>
<td>( (1+i_{T+2}^{r*})S_{T+2}^{r} )</td>
<td>( (1+i_{T+2}^{r*})\Delta_{T+1,T}^{r}+\Delta_{T+2,T+1}^{r} )</td>
<td>( g_{T+2}^{r} )</td>
<td>( f_{T+2}^{r} )</td>
<td>( E_{T+2}^{r} )</td>
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<td>( \vdots )</td>
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<td>( \vdots )</td>
</tr>
<tr>
<td>( T+k )</td>
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<td>( \vdots )</td>
</tr>
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</table>

Before we seek what happens at period \( T+k \), we have to determine how Treasury can finance the interest payments \( f_{T+k}^{r} \) for the receipts, \( E_{T+k}^{r} \). We distinguish between two possible distinct cases and one
intermediate case. Treasury may also finance the interest payments \( f_{T+k}^r \) by borrowing from market and thus repays it and associated interest to market; or may borrow that amount from OG (i.e., state banks) and repays it and associated interest to OG; or tries to do both. We demonstrate both of the distinct cases in sub-sections 5.1 and 5.2, respectively. For the intermediate case, we use the results obtained in 5.1 and 5.2.

### 5.1 Market borrowing to finance the interest payments, \( f_{T+k}^r \)

Following from Appendix B, at any period \( T+k \) if Treasury is forced to pay both last instalment of \( S_T^r \) to OG and associated interests to market, then the additional net-receipt \( E_{T+k}^r \) which Treasury should borrow from market can be given by Eq.(3), which is identical to Eq.(B.7).

\[
E_{T+k}^r = S_T^r \cdot i^{rr} \left( \frac{k-1}{2} \right) + E_T^r \cdot i^r \frac{k(k+1)}{2} + \sum_{j=1}^{k} (1+i^r)^{j-1} \Delta_{T+j, T+j-1}^r \tag{3}
\]

The real interest rate that Treasury guarantees to pay to OG is assumed to be constant, \( i_{T+k}^{rr} = i^{rr} \) for all \( k \). Likewise, the market real ex-post interest rate, \( i_{T+k}^r \) is constant \( i_{T+k}^r = i^r \) for all \( k \). Eq.(3) shows that Treasury’s additional net-borrowing process established in order to totally pay the securitized partition of non-marketable debt stock exhibits nonlinear character. Moreover, if the last term in RHS of Eq.(3) is zero, implying no further hidden transactions during payment process, the process shown by Eq.(3) certainly do not follow stochastic character, but follows only nonlinear character.

### 5.2 Borrowing from OG to finance interest payments, \( f_{T+k}^r \)

Following from Appendix B at any period \( T+k \), if Treasury is forced to pay both last instalment of \( S_T^r \) to OG and associated interests to market, then the additional net-receipt \( E_{T+k}^r \) which Treasury should borrow from market is given by Eq.(4), which is identical to Eq.(B.12).

\[
E_{T+k}^r = \left( \frac{k-1}{2} \right) i^{rr} S_T^r + \sum_{j=1}^{k} (1+i^r)^{j-1} \Delta_{T+j, T+j-1}^r + i^r S_T^r \left[ \frac{(k+1)(k+2)}{3} \right] \left[ 1+i^{rr} \left( \frac{k-1}{4} \right) \right] \tag{4}
\]

Different from the first policy, Treasury finances the interests associated with the receipts \( E_{T+k}^r \), by re-borrowing from OG. When we compare Eq.(3) to Eq.(4) we easily see that the magnitude of illiquidity under second policy is greater than that of first policy. Given two distinct policies, Eq.(3) and Eq.(4) show respectively the time evolution of magnitude of illiquidity, unless it occurs. Therefore if Treasury is forced exogenously or endogenously to complete rest of the stock at some period where the payment process has not
accomplished yet, then from market Treasury should borrow the additional net-amount given by Eq.(3) or Eq.(4) depending upon the financing-source of interest payments associated with that additional net-borrowing from market.

5.3 2000-2001 Crisis

Within 1992-1996 Treasury issued Holding bonds for the debts of SEE, SSI, and EBF (see Dikec 2001). As a result the debts of these OG are holded by Treasury. Moreover, In 1984,1992 and 1999 the debts of LA and SEE to SSIs, State Banks, CBRT and the debts of LA and SEE to each other are consolidated by Treasury and Trasury issued Consolidation G-bonds to creditor public entities (see Dikec 2001). On the other hand Eichengreen (2001) point out that in 1999 duty loses reach %15 of GDP and the main part of these duty loses is unsecuritized. Following from the IMF stabilization programme, a share of non-marketable debt stock is securitized by issuance of non-cash G-Bonds and associated payment process was scheduled for the period 1999-2002 (Dikec 2001). Above argument enables us to accept that Treasury initialized the process in 1999Q1 and considered to finish the securitized non-marketable debt stock within $K=11$ quarters, implying 2001Q4. We choose $k=10$ instead of $k=9$, because in 2001Q2 and 2001Q3 there are also two spikes. Figure 3 depicts the extra net-borrowing $E'_{T+k}$ between 1997Q4-2002Q1 in real terms$^{11}$ and in terms of mTL.

From 1999Q1, $k=10$ periods yields, 2001Q3. The average $E'_{T}$ is computed for interval 1999Q1-2000Q4, the interval before the spike occurs; average $E'_{T}=3,08$. Let us assume that there exists no new hidden transactions, $\Delta'_{T+k,T+k-1}=0$ during $k$-periods. Moreover the real ex-post interest rate$^{12}$ that Treasury had paid to OG is $i''=-0,1669$ and that the market real interest rate$^{13}$ is $i''=0,18$ for all $k$. The first case yields according to Eq.(3) that $\hat{E}'_{2000Q1-Q4}=S'_{T}(-0,75)+30,8$. The second case yields according to Eq.(4) that $\hat{E}'_{2000Q1-Q4}=54,516$. We can determine with probability 1 that, if Treasury was in intermediate case, during [8-11]th periods in order to finish $S'_{T}$ and interest payments associated with its payment process, then

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$^{11}$ The nominal extra net-borrowing values are adjusted by 1987 Consumer Price Index given by Turkish Statistical Association www.tuik.gov.tr.

$^{12}$ In order to obtain intra-gov nominal ex-post interest rate for 1999, the interest payments on non-cash G-bond and T-bill stocks are taken into account and Eq.(4) is applied. Then the nominal ex-post interest rate is adjusted by the annual inflation rate through 1999. The data is obtained from “1999 Treasury Operations Report” released by Turkish Court of Accounts.

$^{13}$ According to the data by CBRT and SPO, annual average compound (fixed) coupon rates on T-Bill and G-Bonds auctions in 1999 are adjusted by annual inflation and converted to quarterly avg rate. www.tcmb.gov.tr.
Treasury should borrow from the market the total amount given by $E_{2001Q1-Q4}^T$, such that, $S_T^r(-0.75) + 30.8 < E_{2001Q1-Q4}^T < 54,516$ holds. However during [8-11]th periods Treasury borrowed respectively, 23,80079 at 2001Q1; 19,79282 at 2001Q2; 11,25539 at 2001Q3, and 2,682516 at 2001Q4, totally equals to 57,53. Therefore, the ordering should be $S_T^r(-0.75) + 30.8 < E_{2001Q1-Q4}^T = 57,53$ implying that Treasury has a strategy as defined by second case. Since we could not directly observe the total amount of non-marketable debt stock, we focus on the payment process of the securitized partition of the non-marketable debt stock. Since Treasury finances the payment process by borrowing from market, the extra amount can be directly observed from the Treasury’s net-borrowing which exceeds the budget financing requirement of the public sector.

**Figure 3** The series $E_{T+k}^r$ between 1997Q4 – 2002Q1

When we compare our findings with the reportings of literature: In a general equilibrium framework Voyvoda and Yeldan (2005) utilized OLG model of growth, and calibrated to Turkish data over 1990s. The authors report that the path of aggregate public debt as a ratio to GNP displays significant degree of inertia and would be brought down only gradually and slowly. However, we show that when non-marketable debt stock is taken into account, the gradual bringing down of debt stock is resulted by liquidity crisis.

6. Non-standard approach to additional net-borrowing data

This sub-section introduces the methodology that we use for phase-space analysis of the data for extra net-borrowing in real terms observed within period 1989Q1-2009Q4 and finally presents the results. Let us denote the dynamical system, $f : R^n \rightarrow R^n$, with the trajectory,
\[ x_{t+1} = f(x_t) + \varepsilon_t, \ t = 0, 1, 2, \ldots \]  \hspace{1cm} (5)

Associated with the dynamical system in Eq.(5) there is a measurement function \( h : \mathbb{R}^n \rightarrow \mathbb{R} \) which generates the time series, \( z_t = h(x_t) \). It is assumed that all that is available to observer is the sequence \( \{z_t\} \).

The dynamical system itself may be assumed to be contaminated by noise, or the observed time series \( z_t \) may be assumed to convey noise\(^{14}\). Following Takens’ theorem (Takens 1981), from observed time series \( \{z_t\} \), one can generate the data vector \( y_i = (z_i, z_{i+d}, \ldots, z_{i+(m-1)d}) \) for all \( i \in (N-(m-1)d) \) where \( N \) is the length of the observed sequence \( \{z_t\} \); \( d \) is the time delay. This vector indicates a point of \( m \)-dimensional reconstructed phase space \( \mathbb{R}^m \), where \( m \) is embedding dimension. The reconstructed trajectory is an embedding of the original trajectory when the \( m \) value is sufficiently large, \( m \geq 2n+1 \), upper-worst case.

Depending on the data, embedding can be established even when \( m \) is less than \( 2n+1 \) (Gencay Dechert 1992). Abarbanel (1995) give us a good suggestion on how to select \( m \) and \( d \). From now on the time delay \( d \), is taken to be equal to 1, which corresponds our observation interval in time domain. First of all, we reconstruct the appropriate phase-space for extra-net borrowing data in real terms by \( E'_r \). Figure 3 depicts the orbit of additional net-borrowing process in real terms, \( E'_r \). The \( x \)-axis (ERx) shows \( E'_{r+1} \); \( y \)-axis (ERy) shows \( E'_{r+1} \) and \( z \)-axis (ERz) shows \( E'_{r+2} \), according to the embedding dimension\(^{15} \), \( m = 3 \). The points which are located extremely far from the attractor correspond to the spikes observed in time domain.

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\(^{14}\) Kantz’s algorithm allows us to make this assumption, Kantz (1994).

\(^{15}\) The embedding dimension is computed to be \( m = 3 \) or 4. Figure 3 is drawn for 3-dimension.
To compute maximal Lyapunov exponent directly, similar to the algorithm of Wolf et al. (1985), Kantz (1994) use the fact that the distance between two trajectories typically increases with a rate given by the max LE in $\mathbb{R}^n$. Therefore we define the distance between a reference trajectory $y_t$ and one of its $\varepsilon$ – neighbor(s) $y_t^{(e)}$ after the relative time (iteration) $\tau$ by a function:

$$D(): \mathbb{R}^n \to \mathbb{R} \quad \text{and} \quad D(y_t, y_t^{(e)}; \tau) = |y_{t+\tau} - y_{t+\tau}^{(e)}|$$

Eq.(6) gives the magnitude of the difference vector $(y_{t+\tau} - y_{t+\tau}^{(e)})$ lying between the point $y_{t+\tau}$ and the point $y_{t+\tau}^{(e)}$. The logarithm of $D(.)$ is needed to smooth the output of the function. We compute for all $t=1,2,...,T$

where $T$ is the number of reference points$^{16}$ on the orbit. Thus we obtain

$$S(\tau) = \frac{1}{T} \sum_{t=1}^{T} \ln \left( \frac{1}{|U_t|} \sum_{y_t^{(e)} \in U_t} D(y_t, y_t^{(e)}; \tau) \right)$$

where $|U_t|$ denotes the number of elements of set of $\varepsilon$ – neighbors of $y_t$. Finally, the slope of the curve $S(\tau)$ gives us the maximal Lyapunov exponent a la Kantz:

$$\frac{\partial S(\tau)}{\partial \tau} \approx \lambda_{\text{max}}(t)$$

for any $\tau$ in the scaling region,

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$^{16}$ Reference point is the point on the orbit which has at least one $\varepsilon$ - neighbor.
\[ \tau \leq \tau_{\text{max}}. \] In summary, our numerical value for the maximal Lyapunov exponent is the slope of the curve \( S(\tau) \) in the scaling region. The results of the approach\(^{17}\) are depicted in Figure 4.

**Figure 4.** The time evolution of initially nearby points - average log distances for \( E'_T \)

Figure 4 plots the output of Eq.(7), \( x \)-axis shows the number of iterations, \( \tau \) while \( y \)-axis shows the average log distances, \( S(\tau) \). In Figure 4, the average slope of the curves gives us the maximal LE, \( \lambda_{\text{max}}(t) \) a la Kantz (1994). The curves differ from each other in sense of the embedding dimension (\( m \)) and \( \varepsilon \) chosen (eps). The \( \tau \) value(s) where the slope of the curves approximates zero is denoted by \( \tau_{\text{max}} \) and signifies the last step of scaling range, implying that the dynamical system is still stable. According to Figure 4, the slope of the curves is positive implying that the data of extra net-borrowing in real terms exhibit chaotic process, with \( \tau_{\text{max}} = 9 \).

The amount of additional net-borrowing can be predictable for 9 lags and then the payment process becomes unstable, implying an inevitable debt crisis. In words, if the number of scheduled payments exceeds 9 lags, then we expect to observe an eventual spike in time-domain, implying that this transition is inevitable. We also conclude that the velocity of the divergence confirms the findings of Özkan (2005). The author states that given the gap between the public sector borrowing requirement and the size of the domestic capital markets the outcome was ever-increasing real interest rates on domestic borrowing, which, in turn, became the source of further deterioration in public balances.

\(^{17}\)Tisean Package introduced by Hegger and Kantz (1999) is used to compute Eq.(7).
7. Results and Discussion

In this paper we analyzed the public sector debt dynamics in different perspective. In contrast with (Akyüz and Boratav 2003), we showed that the non-marketable debt stock was not the outcome of IMF policies, instead it was in play in 2000–2001 Turkish crisis episodes. Our study contributes the literature in various respects. First of all, it is understood that even though Turkish economy was under the surveillance of the IMF due to the stabilization programme, Turkish authorities has misled the IMF staff regarding the hidden public debt stock and invisible budget process. Secondly, we introduced informal debt-creating transactions, which will strengthen the efforts on fiscal transparency. We established a general model for the accumulation of non-marketable debt stock and explained the high magnitude spikes observed in data. Applying findings of the model to the obtained data, we showed that the magnitudes of illiquidity can be explained by our model. Finally, we applied “non-standard” approach to the data of Treasury’s payment process and checked the explicative power of our theoretical model. The problem in most of the studies that analyze public debt sustainability (standard approach) in the aftermath of the global finance crisis is that, they suffer from not taking into account non-linearity: The crisis period may be exceptional in the sense that the fiscal stimulus packages put heavy burden on the public debt while the low economic growth rates lead to further deteriorations of the debt dynamics. In this sense, debt dynamics may follow a non-linear path.

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APPENDIX A

Total budget balance = Budgetary balance + Non-budgetary balance (A.1)

Budgetary balance = Total budget revenues – Total budget expenditures (including interest payments) (A.2)

Non-budgetary balance = Total non-budgetary receipts – Non-budgetary and Securitized payments (A.3)

Balance of non-budgetary and Securitized transactions = \{ Non-budgetary and Securitized receipts - Non-budgetary and Securitized payments \} (A.4)

Eq.(4) shows us what Treasury announces as “net-increase in non-cash debt stock”.

Balance of non-budgetary and Unsecuritized transactions = \{ net Non-budgetary and Unsecuritized receipts \} (A.5)

Net-increase in marketable debt stock = - (Budgetary balance) + (Non-marketable debt stock principal payments) (A.6)

By the definition of non-budgetary transactions, Eq. (A.7) holds.

Net-increase in non-marketable public debt stock = \{ Non-budgetary balance - Non-budgetary and Unsecuritized payments \} (A.7)

Non-marketable debt stock principal payments = \{ Non-budgetary and Securitized payments + \} (A.8)
Non-budgetary and Unsecuritized payments

"Accurate" net-increase in total public debt stock = \{ Net-increase in marketable debt stock \\
+ Net-increase in non-marketable debt stock \} \ (A.9)

"Accurate" borrowing requirement of public sector = "Accurate" net-increase in total public debt stock \ (A.10)

"Accurate" net-increase in public debt stock = \{ Net-increase in marketable debt stock + Non-budgetary balance in public debt stock \} - Non-budgetary and Unsecuritized payments \ (A.11)

Finally, by replacing Eq.(A.6) into above equation we obtain Eq.(A.11).

"Announced" borrowing requirement of public sector (PSBR) is shown by Eq. (A.12).

"Announced" borrowing requirement of public sector (PSBR) = - (Budgetary balance) \ (A.12)

APPENDIX B

Section 5.1.
At the period \( T+k \), the result of the dynamical process \( \Omega \) yields the real non-marketable debt stock, which is given by Eq.(B.1).

\[
S_r^r \left( \prod_{j=1}^k \left( 1 + \bar{i}_{r+k}^r \right) \right) - \sum_{j=1}^k g_{T+k-j}^r \prod_{l=1}^j \left( 1 + \bar{i}_{T+k-l}^r \right) + \sum_{j=1}^{k-1} \Delta_{T+k-j,T+k-j-1}^r \prod_{l=1}^j \left( 1 + \bar{i}_{T+k-l}^r \right) + \Delta_{T+k,T+k-1}^r \ (B.1)
\]

and the additional net amount to be borrowed from market at period \( k \), \( E_{T+k}^r \) is obtained as Eq.(B.2).

\[
E_{T+k}^r = g_{T+k}^r + \sum_{j=1}^k E_{T+k-j}^r \prod_{l=1}^j \left( 1 + \bar{i}_{T+k-l}^r \right) - \sum_{j=1}^k E_{T+j-1}^r \ (B.2)
\]
The first term in RHS of Eq.(B.2) denotes the \((k+1)h\) payment for \(S'_k\). The term in brackets in Eq.(B.2) shows the accumulated effect of the interest rates, \(f^{r}_{T+k}\) on receipts \(E^{r}_{T+k}\). The interest rate on the debt stock among governmental institutions are determined by consensus and by hierarchial relations (TCA reports 1999). Therefore, in Eq.(B.1) we may assume that the real interest rate that Treasury guarantees to pay to OG is constant, \(i^{r*}_{T+k} = i^{r*}\) for all \(k\). Therefore, at period \(T+k\) the real non-marketable debt stock to be paid is shown by Eq.(B.3).

\[
\left(1 + i^{r*}\right) S'_k = \sum_{j=1}^{k} \left(1 + i^{r*}\right)^{j-1} g^{r}_{T+j} + \sum_{j=1}^{k} \left(1 + i^{r*}\right)^{k-j} \Delta'_{T+j, T+j-1} \tag{B.3}
\]

Without no loss of generality we may assume that Treasury prefers to pay the non-marketable stock in \(j\) instalments (equally divided amounts) : \(g^{r}_{T+j} = g^{r}_{T+j} \) for all \(1 \leq j < k\). If we replace these values into Eq.(B.3), we obtain the stock at period \(T+k\) as given in Eq.(B.4).

\[
\left(1 + i^{r*}\right) S'_k = \sum_{j=1}^{k} \left(1 + i^{r*}\right)^{j} g^{r}_{T+j} + \sum_{j=1}^{k} \left(1 + i^{r*}\right)^{k-j} \Delta'_{T+j, T+j-1} \tag{B.4}
\]

If Treasury is forced to pay the rest of stock at period \(T+k\), then Treasury should finance the amount given in Eq.(B.4) during period \(T+k\). Therefore the amount given in Eq.(B.4) can be considered as the last instalment of the debt stock, and naturally can be denoted by \(g^{r}_{T+k}\).

By using the realtion \((1+c)^k \to (1+kc)\) for \(0 \leq c << 1\) and \(k > 1\), Eq.(B.4) yields

\[
g^{r}_{T+k} = \frac{k-1}{2} i^{r*} S'_k + \sum_{j=1}^{k} \left(1 + i^{r*}\right)^{k-j} \Delta'_{T+j, T+j-1} \tag{B.5}
\]

On the other hand, Treasury should also finance the cost-of-the additional net borrowing, \(f^{r}_{T+k}\). To determine the amount of cost-of-the additional borrowing at period \(T+k\), first assume that the market real ex-post interest rate, \(i^{r*}_{T+k}\) is constant \(i^{r*}_{T+k} = i^{r*}\) for all \(k\). Thus Eq.(B.2) is rewritten as Eq.(B.6).

\[
E^{r}_{T+k} = g^{r}_{T+k} + \sum_{j=1}^{k} \left(1 + i^{r*}\right)^{k-j+1} E^{r}_{T+j-1} - \sum_{j=1}^{k} E^{r}_{T+j-1} \tag{B.6}
\]

Second, let us assume that the receipts in real values are equal to each other in every period \(j, \ j < k\), implying \(E^{r}_{T+j} = E^{r}_{T}\) for all \(1 \leq j < k\). Therefore at any period \(T+k\), if Treasury is forced to pay both last instalment of \(S'_T\) to OG and associated interests to market, then the extra net-receipt \(E^{r}_{T+k}\) which Treasury should borrow from market can be given by the equation

\[
E^{r}_{T+k} = S'_T i^{r*} \frac{k(k+1)}{2} + E^{r}_{T} i^{r*} \frac{k(k+1)}{2} + \sum_{j=1}^{k} \left(1 + i^{r*}\right)^{k-j} \Delta'_{T+j, T+j-1} \tag{B.7}
\]

Section 5.2
At the period $T+k$, the result of the dynamical process $\Omega$ yields the real debt stock,

$$
\sum_{j=1}^{k} S_{T}'(1+i'^{r}_{j}) - \left[ \sum_{j=1}^{k} g_{T+k-j}^{r} \prod_{l=1}^{j}(1+i'^{r}_{T+k-l+1}) \right] + \left[ \sum_{j=1}^{k} \Delta r_{T+k-j,T+k-j-1}^{r} \prod_{l=1}^{j}(1+i'^{r}_{T+k-l+1}) \right] + \Delta r_{T+k,T+k-1}^{r}
$$

$$
+ \sum_{j=1}^{k-1} f_{T+k-j}^{r} \prod_{l=1}^{j}(1+i'^{r}_{T+k-l+1}) + f_{T+k}^{r}
$$

(B.8)

where the sum of last two terms in Eq.(B.8) shows the accumulated effect of the interest for receipts, $E_{T+k}'$ from market. The sum of these two terms is more complicated since Treasury finances the interest payments of $E_{T+k}'$ by borrowing from OG. Therefore, this sum is affected by both the real interest rate paid to OG and real interest rate paid to market. Thus at period $T+k$, the particular interest payment can be computed by Eq.(B.9).

$$
f_{T+k}' = \sum_{j=1}^{k} E_{T+k-j}^{r} \prod_{l=1}^{j}(1+i'^{r}_{T+k-l+1}) - \sum_{j=1}^{k} E_{T+j-1}^{r}
$$

(B.9)

As we have done, assume that the real interest rate that Treasury guarantees to pay to OG is constant $i'^{r}_{T+k} = i^r$ and that the market’s real ex-post interest rate, $i'^{r}_{T+k}$ is constant $i'^{r}_{T+k} = i^r$ for all $k$. Replacing Eq.(B.9) into the last two terms in RHS of Eq.(B.8) yields total interest payments as given in Eq.(B.10). That is,

$$
\sum_{j=1}^{k-1} f_{T+k-j}^{r} \prod_{l=1}^{j}(1+i'^{r}_{T+k-l+1}) + f_{T+k}^{r} = \frac{i^r.E_{r}^{T}}{2} \left[ \frac{k(k+1)(k+2)}{3} \right] \left[ 1+i'^{r}\left( \frac{k-1}{4} \right) \right]
$$

(B.10)

As it can be seen from Eq.(B.10), the total interest payments are consisted of interest payments for $E_{T+k}'$ to market and interest payments for cost-of-the additional borrowing ( $f_{T+k}'$) to be paid to OG. If we replace Eq.(B.10) in place of last two terms in Eq.(B.8) and complete the algebraic operations, we obtain the real non-marketable debt stock at period $T+k$ as given in Eq.(B.11).

$$
\left( \frac{k-1}{2} \right) i'^{r} S_{T}' + \sum_{j=1}^{k} (1+i^r) \Delta r_{T+j,T+j-1}^{r} + \frac{i^r.E_{r}^{T}}{2} \left[ \frac{k(k+1)(k+2)}{3} \right] \left[ 1+i'^{r}\left( \frac{k-1}{4} \right) \right]
$$

(B.11)

Since Treasury borrows from OG the amount to pay interests associated with the receipts $E_{T+k}'$, without no loss of generality we may assume that each instalment which is paid to OG is equal to the additional net-borrowing from market: That is $E_{T+k}' = g_{T+k}'$ for all $k$. If Treasury is forced to finish the rest of stock $S_{T}'$ at period $T+k$, then Treasury should finance the amount given in Eq.(B.11) during the period $T+k$. Therefore, the amount given in Eq.(B.11) can be considered as the last instalment of the debt stock, and naturally can be denoted by $g_{T+k}'$. Since Treasury’s additional net-borrowing from market $E_{T+k}'$ should be identical to the last instalment of $S_{T}'$, $g_{T+k}'$, thus Eq.(B.12) holds.

$$
E_{T+k}' = \left( \frac{k-1}{2} \right) i'^{r} S_{T}' + \sum_{j=1}^{k} (1+i^r) \Delta r_{T+j,T+j-1}^{r} + \frac{i^r.E_{r}^{T}}{2} \left[ \frac{k(k+1)(k+2)}{3} \right] \left[ 1+i'^{r}\left( \frac{k-1}{4} \right) \right]
$$

(B.12)